Game Theory — Mock Exam

Enrico Mattia Salonia

Academic Year 2025–2026

Time: 75 minutes. Be clear and concise.

Part I — Definitions and True/False

Question 1 (4 points)**.** Define an ordinal game in strategic form.

Question 2 (7 points)**.** Say whether the following statements are true, false or uncertain and *justify* your answer. ('uncertain' means that the statement is true under conditions that are not explicitly mentioned in it)

(a) **[3.5 points]** Consider the following strategic-form game.

	L	R
U	$\begin{pmatrix} o_1 & o_2 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	o_3
D	o_4	$\begin{pmatrix} o_3 & o_4 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Players' preferences over outcomes are:

$$o_1 \succ_1 o_3 \succ_1 o_4 \succ_1 o_2, \qquad o_4 \succ_2 o_1 \succ_2 o_3 \succ_2 o_2.$$

Statement: "There is a Nash equilibrium".

(b) **[3.5 points] Statement:** "The following extensive form has two subgames, of which only one is a proper subgame (i.e., not the whole game)".

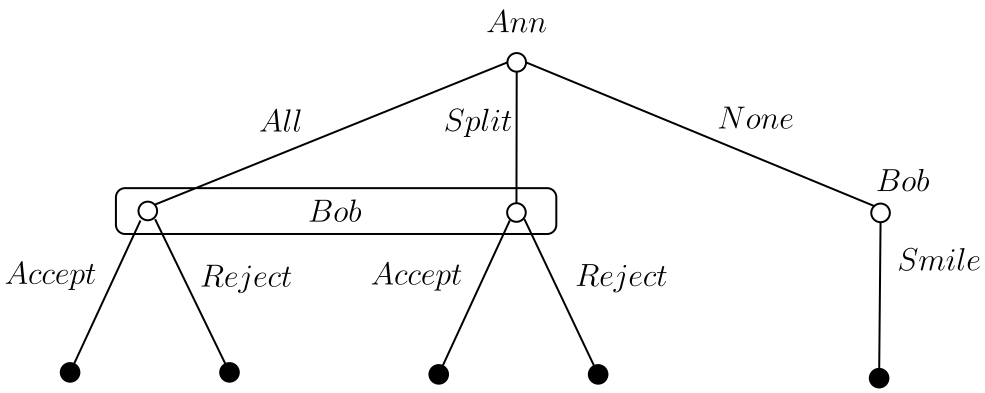

Figure 1: Extensive-form game for statement (b).

Part II — Exercises

Question 3 (7 points)**.** Consider the following strategic-form game with two players who have expected utility preferences:

	L	M	R
A	$(2,1)$	$(0,2)$	$(1,0)$
B	$(0,2)$	$(2,1)$	$(1,0)$
C	$(0,0)$	$(0,0)$	$(0,3)$

1. Perform the Cardinal Iterated Deletion of Strictly Dominated Strategies.

2. Find all Nash equilibria in mixed strategies of the game.

Question 4 (7 points)**.** Consider the following trust game. Ann has 10 euros and moves first:

- if she chooses **Keep** (K), she keeps her 10 euros and Bob gets nothing;
- if she chooses **Send** (S), Bob receives 30 euros.

After S, Bob chooses a return $b \in \{0, 10, 20, 30\}$ to Ann. Bob's preferences are determined by Nature *before* the game begins but they are *not observed* by Ann. With probability $p \in (0, 1)$ Bob is **generous**: a Beckerian altruist with utility $U_B = m_B + 2\,m_A$. With probability $1 - p$ Bob is **selfish**: he maximizes m_B only. Ann is selfish and maximizes her *expected* monetary payoff. She knows p but not Bob's realized preferences.

1. Find the optimal return b^* for each of Bob's preferences after S.
2. Compute Ann's expected payoff from S and from K as a function of p.
3. Find the threshold p^* such that Ann chooses S if and only if $p > p^*$.
4. Briefly interpret p^*: what must Ann believe about the population of potential Bobs for trust to emerge?

Part III — Advanced Question

Question 5 (6 points)**.** Two energy companies, Firm 1 and Firm 2, compete for drilling rights to a newly discovered oil field. Each firm submits one sealed bid without observing the other bid. The higher bidder wins the concession and pays exactly its own bid. If bids are equal, each firm wins with probability $\frac{1}{2}$. Both firms have expected utility preferences.

The oil field has a common economic value that can be $V = 0$ or $V = 100$. Both firms believe:

$$p(V = 100) = \tfrac{1}{2}, \qquad p(V = 0) = \tfrac{1}{2}.$$

Bids are restricted to three pure strategies $\{40, 60, 80\}$.

1. Compute each firm's expected value of the oil field.
2. Build the expected-payoff table (Firm 1 rows, Firm 2 columns).
3. Find all pure-strategy Nash equilibria.
4. At each pure-strategy equilibrium, does the winning firm pay more than the expected value of the oil field?
5. Now suppose that a regulatory constraint removes bid 40 from the feasible set, so that each firm can only bid from $\{60, 80\}$. Find the pure-strategy Nash equilibrium of this reduced game and check again whether the winner pays more than the expected value.

Solutions

Solution to Question 1. An ordinal game in strategic form is a list

$$\langle I, (S_1, \ldots, S_n), O, f, (\succsim_1, \ldots, \succsim_n) \rangle,$$

where:

- $I = \{1, 2, \ldots, n\}$ is the set of players;
- $(S_1, S_2, \ldots, S_n)$ is the list of strategy sets;
- O is the set of outcomes;
- $f : S \to O$ maps each strategy profile $s \in S = S_1 \times \cdots \times S_n$ to an outcome $f(s) \in O$;
- for each player i, $\succsim_i$ is a preference over outcomes.

Solution to Question 2.

(a) **Uncertain.** The statement concerns equilibrium existence, but some strategy profiles lead to lotteries and only ordinal preferences over outcomes are provided. Without knowing preferences over lotteries one cannot determine equilibrium existence from the given data alone.

(b) **True.** The two Bob nodes connected by one information set cannot start a subgame. The only proper subgame starts at the singleton Bob node on the right branch. Therefore subgames are: (i) the whole game, and (ii) that right-branch proper subgame.

Solution to Question 3.

(1) For Player 1, strategy C is strictly dominated by the mixed strategy

$$\sigma_1 = \begin{pmatrix} A & B & C \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix},$$

against L, M, R, this mixture gives payoff $1, 1, 1$, while C gives $0, 0, 0$. Eliminate C.

In the reduced game $\{A, B\} \times \{L, M, R\}$, Player 2 strategy R is strictly dominated by M.

The game reduces to:

	L	M
A	$(2, 1)$	$(0, 2)$
B	$(0, 2)$	$(2, 1)$

(2) No pure Nash equilibria. Let Player 1 play A with probability p, Player 2 play L with probability q. Indifference conditions are:

$$2q = 2(1 - q) \Rightarrow q = \tfrac{1}{2}, \qquad (2 - p) = (1 + p) \Rightarrow p = \tfrac{1}{2}.$$

Hence the unique Nash equilibrium is

$$\sigma = (\sigma_1, \sigma_2), \quad \sigma_1 = \begin{pmatrix} A & B & C \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} L & M & R \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}.$$

Solution to Question 4.

(1) Preference-contingent optimal return after S:

- **Generous type** ($U_B = m_B + 2m_A$):

$$U_B = (30 - b) + 2b = 30 + b,$$

so U_B is increasing in b, hence $b_G^* = 30$.

- **Selfish type** ($U_B = m_B$):

$$U_B = 30 - b,$$

so U_B is decreasing in b, hence $b_S^* = 0$.

(2) Ann's expected payoff:

$$\mathbb{E}[m_A \mid S] = p \cdot 30 + (1 - p) \cdot 0 = 30p, \qquad m_A(K) = 10.$$

(3) Ann chooses S iff

$$30p > 10 \iff p > \tfrac{1}{3}.$$

Therefore the threshold is

$$p^* = \tfrac{1}{3}.$$

At $p = \frac{1}{3}$, Ann is indifferent between K and S.

(4) Interpretation: trust emerges only if Ann believes the probability of meeting a generous Bob is sufficiently high. Here she must believe that at least one third of potential partners are generous.

Solution to Question 5.

(1) Each firm's expected value is

$$\mathbb{E}[V] = \tfrac{1}{2} \cdot 100 + \tfrac{1}{2} \cdot 0 = 50.$$

(2) If a firm wins with bid b, its expected payoff is $50-b$. If both bid b, each wins with probability $\frac{1}{2}$ and gets $\frac{1}{2}(50 - b)$. The expected-payoff matrix is:

	40	60	80
40	$(5,\ 5)$	$(0,\ -10)$	$(0,\ -30)$
60	$(-10,\ 0)$	$(-5,\ -5)$	$(0,\ -30)$
80	$(-30,\ 0)$	$(-30,\ 0)$	$(-15,\ -15)$

(3) Best responses:

- Against 40: best response is 40 (payoff $5 > -10 > -30$).
- Against 60: best response is 40 (payoff $0 > -5 > -30$).
- Against 80: best response is 40 (payoff $0 = 0 > -15$); 60 is also a best response.

By symmetry the same holds for both firms. The unique pure-strategy Nash equilibrium is $(40, 40)$.

(4) At $(40, 40)$ each firm wins with probability $\frac{1}{2}$ and pays 40. Since $40 < \mathbb{E}[V] = 50$, the winner does *not* pay more than the expected value. Firms bid conservatively precisely to avoid overpaying.

(5) In the reduced game $\{60, 80\}$:

	60	80
60	$(-5,\ -5)$	$(0,\ -30)$
80	$(-30,\ 0)$	$(-15,\ -15)$

Best response to 60 is 60; best response to 80 is 60. The unique pure-strategy equilibrium is $(60, 60)$. The winner pays $60 > 50 = \mathbb{E}[V]$.

Comment. When firms can bid below the expected value, competition drives bids down and the curse is avoided. Removing the low bid forces both firms into bids that exceed the expected value, and the winner necessarily overpays. This illustrates how restricted competition or bidding floors can induce the winner's curse even when firms would otherwise avoid it.

Game Theory — Exam - A

Enrico Mattia Salonia

2 April 2026

Time: 75 minutes. Be clear and concise.

Part I — Definitions and True/False

Question 1 (4 points). Define a Nash Equilibrium strategy profile in an ordinal game in strategic form with n players.

Question 2 (7 points). Say whether the following statements are true, false, or uncertain, and *justify* your answer. ("uncertain" means that the statement is true under conditions that are not explicitly mentioned.)

(a) [**3.5 points**] Consider the following game: Ann has an apple, while Bob has a banana. They simultaneously choose whether to give their fruit to the other or to keep it. **Statement:** "Each player in this game has a strictly dominant strategy."

(b) [**3.5 points**] **Statement:** "In the following extensive-form game with imperfect information, each player has two pure strategies."

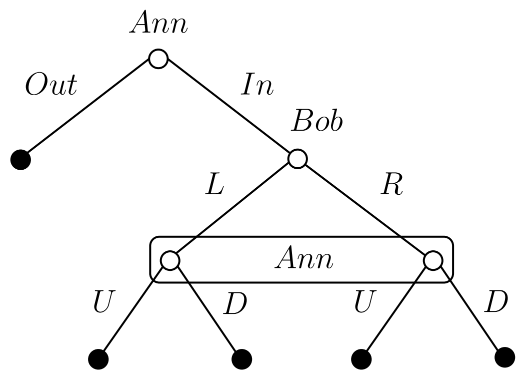

Figure 1: Extensive-form game for statement (b).

Part II — Exercises

Question 3 (7 points). Consider the following strategic-form game with two players who have expected utility (von Neumann-Morgenstern) preferences:

	L	M	R
A	$(8, 1)$	$(2, 7)$	$(5, 0)$
B	$(1, 8)$	$(7, 2)$	$(5, 0)$
C	$(1, 3)$	$(2, 3)$	$(0, 9)$

(a) Perform the Cardinal Iterated Deletion of Strictly Dominated Strategies.

(b) Find all Nash equilibria in mixed strategies of the game.

Question 4 (7 points). Firm 2 is an incumbent monopolist earning a profit of 2 (million euros). Firm 1 is a potential entrant that currently earns a profit of 2 in another market. Firm 1 moves first and decides whether to *Enter* this market or *Stay Out*.

- If Firm 1 stays out, both firms keep their current profits.
- If Firm 1 enters, Firm 2 observes the entry and chooses whether to *Fight* (start a price war) or *Accommodate* (share the market).

– If Firm 2 fights, the price war drives Firm 1's profit to 0 and Firm 2's profit to 1.
– If Firm 2 accommodates, Firm 1 earns 4 and Firm 2 earns 3.

Assume that for both firms payoffs equal profits.

(a) Draw the tree of the extensive-form game with perfect information.

(b) Write the strategic form of the game and find all pure-strategy Nash equilibria.

(c) Solve the game by backward induction and find all the subgame-perfect equilibria.

(d) Are there Nash equilibria that are not subgame-perfect? If yes, why?

Part III — Advanced Question

Question 5 (6 points)**.** Three students must decide whether to collectively hire a private tutor for a game theory exam review session. The cost of the tutor would be split equally, so $c_1 = c_2 = c_3 = 15$. For every student $i = 1, 2, 3$, let v_i be the gross benefit from having the tutor hired. The gross benefits are $v_1 = 25$, $v_2 = 20$, and $v_3 = 5$. The net benefit to student i is $v_i - c_i$. Student i has the following utility of wealth function (where m_i denotes student i's wealth):

$$U_i(\$m_i) = \begin{cases} m_i & \text{if the tutor is not hired,} \\ m_i + v_i & \text{if the tutor is hired.} \end{cases}$$

Assume all students are rich enough to afford any cost.

The pivotal mechanism. The students use the following procedure to decide whether to hire the tutor.

Step 1. Each student i simultaneously announces a number w_i, interpreted as their gross benefit from having the tutor hired.

Step 2. Compute $D = \sum_{i=1}^{3}(w_i - c_i)$. If $D > 0$ the tutor is hired; if $D \leq 0$ the tutor is not hired.

Step 3. Pivotality. Student i is said to be *pivotal* if removing her announcement would reverse the decision. Formally, let $D_{-i} = \sum_{j \neq i}(w_j - c_j)$. Student i is pivotal if the sign of D_{-i} is opposite to the sign of D (or if $D_{-i} = 0$ and $D \neq 0$).

Step 4. Tax. A pivotal student i pays a tax equal to $|D_{-i}|$; a non-pivotal student pays no tax. The tax is paid on top of the cost share c_i if the project is carried out.

(a) What is the Pareto-efficient decision: to hire the tutor or not?

(b) As you know, in the pivotal mechanism each student has a weakly dominant strategy. If all the students played their weakly dominant strategies, would the tutor be hired? Who is pivotal and what tax does each student pay?

(c) Assume now that the students try to fund the tutor through *voluntary contributions*. Each student i simultaneously chooses a contribution $c_i \in [0, e]$, where e is the student's budget. The amount collected is used to improve the quality of the tutoring session, benefiting everyone. Payoffs are

$$\pi_i(c_i, c_{-i}) = e - c_i + r\sum_{j=1}^{3} c_j, \qquad \tfrac{1}{3} < r < 1.$$

Explain briefly: What is each student's dominant strategy in this voluntary contribution game? Is the resulting equilibrium Pareto efficient? Can you now give a rationale for using the pivotal mechanism studied in parts (a)–(b)?

Solutions

Solution to Question 1. Given an ordinal game in strategic form with n players,

$$\langle I, (S_1, \ldots, S_n), O, f, (\succsim_1, \ldots, \succsim_n)\rangle,$$

a strategy profile $s^* = (s_1^*, \ldots, s_n^*) \in S_1 \times \cdots \times S_n$ is a Nash equilibrium if, for every player $i = 1, \ldots, n$,

$$\pi_i(s^*) \geq \pi_i(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*) \quad \text{for all } s_i \in S_i.$$

Equivalently, each player's strategy in s^* is a best response to the strategies of the other players; no player can gain from a unilateral deviation.

Solution to Question 2.

(a) **Uncertain.** The statement "Each player has a strictly dominant strategy" depends on preferences, which are not specified. If each player only cares about receiving the other fruit (and not about giving away their own fruit), then "Give" is strictly dominant for both players. But if a player dislikes giving away their fruit (or values keeping it enough), "Give" need not dominate "Keep." Therefore, without explicit payoff numbers or preference assumptions, the statement cannot be determined.

(b) **False.** Bob has one decision node (reached after In), so Bob has exactly two pure strategies: L and R.

Ann instead has two information sets (the initial node and the information set grouping the nodes reached after L and R), so a pure strategy must specify one action at each of these information sets. Therefore Ann has four pure strategies: $(In, U), (In, D), (Out, U), (Out, D)$.

Solution to Question 3.

(a) For Player 1, strategy C is strictly dominated by the mixed strategy

$$\sigma_1 = \begin{pmatrix} A & B & C \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix},$$

since against L, M, R, this mixture gives payoffs $4.5, 4.5, 5$, while C gives $1, 2, 0$. So eliminate C.

In the reduced game $\{A, B\} \times \{L, M, R\}$, strategy R for Player 2 is strictly dominated by the pure strategy L, because against A, L gives payoff $1 > 0$, and against B, L gives payoff $8 > 0$. So eliminate R.

The surviving game is

	L	M
A	$(8, 1)$	$(2, 7)$
B	$(1, 8)$	$(7, 2)$

(b) No pure equilibrium exists in the reduced 2×2 game.

Let Player 1 play A with probability p, and Player 2 play L with probability q. Indifference conditions are:

$$8q + 2(1-q) = 1q + 7(1-q) \iff q = \tfrac{5}{12},$$
$$1p + 8(1-p) = 7p + 2(1-p) \iff p = \tfrac{1}{2}.$$

Therefore the mixed-strategy Nash equilibrium is

$$\sigma_1 = \begin{pmatrix} A & B & C \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} L & M & R \\ \frac{5}{12} & \frac{7}{12} & 0 \end{pmatrix}.$$

Solution to Question 4.

(a) The tree has Firm 1 at the root choosing *Enter* or *Stay Out*. The branch *Stay Out* ends with payoff $(2, 2)$. If *Enter*, Firm 2 chooses *Fight* (payoff $(0, 1)$) or *Accommodate* (payoff $(4, 3)$).

(b) Strategic form:

	Fight	*Accommodate*
Enter	$(0, 1)$	$(4, 3)$
Stay Out	$(2, 2)$	$(2, 2)$

Best responses imply two pure-strategy Nash equilibria:

$$(\textit{Stay Out}, \textit{Fight}), \qquad (\textit{Enter}, \textit{Accommodate}).$$

(c) Backward induction: at Firm 2's node, *Accommodate* is optimal since $3 > 1$. Anticipating this, Firm 1 chooses *Enter* since $4 > 2$. Hence the unique subgame-perfect equilibrium is

$$(\textit{Enter}, \textit{Accommodate}).$$

(d) The equilibrium (*Stay Out*, *Fight*) relies on a non-credible threat: "if Firm 1 enters, Firm 2 would fight." This is not credible because in the subgame after *Enter*, Firm 2 strictly prefers *Accommodate* to *Fight* $(3 > 1)$.

Solution to Question 5.

(a) We check whether the sum of net benefits is positive:

$$\sum_{i=1}^{3}(v_i - c_i) = (25 - 15) + (20 - 15) + (5 - 15) = 10 + 5 - 10 = 5 > 0.$$

Since the total net benefit is positive, it is **Pareto efficient to hire the tutor**.

(b) In the pivotal mechanism, the weakly dominant strategy is truthful reporting: $w_i^* = v_i$ for each student.

Under truthful reporting:

$$\sum_{i=1}^{3}(v_i - c_i) = 10 + 5 - 10 = 5 > 0.$$

so the tutor is hired.

Pivotality under truthful play (hiring case):

- Without Student 1: $5 + (-10) = -5 < 0 \;\Rightarrow$ **Pivotal**, tax $= 5$.
- Without Student 2: $10 + (-10) = 0$, and $D = 5 \neq 0 \;\Rightarrow$ **Pivotal**, tax $= |0| = 0$.
- Without Student 3: $10 + 5 = 15 > 0 \;\Rightarrow$ Not pivotal, tax $= 0$.

Student	1	2	3
Pivotal?	Yes	Yes	No
Tax	5	0	0

(c) Student i's payoff in the voluntary contribution game is $\pi_i = e + (r - 1)c_i + r\sum_{j \neq i} c_j$. Since $r < 1$, payoff is decreasing in own c_i, so each student's dominant strategy is $c_i = 0$.

Hence the unique equilibrium is $(0, 0, 0)$, which is not Pareto efficient, because aggregate welfare increases with total contributions when $3r - 1 > 0$ (here $r > \frac{1}{3}$).

This free-rider problem motivates mechanisms such as the pivotal mechanism, which align private incentives with efficient provision of the public good.

Game Theory — Exam - A

Enrico Mattia Salonia

8 June 2026

Time: 75 minutes. Be clear and concise.

Part I — Definitions and True/False

Question 1 (4 points)**.** Define an extensive form with perfect information. What do you need to add to get an extensive form **game** with perfect information?

Question 2 (7 points)**.** Say whether the following statements are true, false, or uncertain, and *justify* your answer. ("uncertain" means that the statement is true under conditions that are not explicitly mentioned.)

(a) **[3.5 points]** Consider the following strategic-form game where the two players have expected utility preferences.

	H	T
H	$(1,-1)$	$(-1,1)$
T	$(-1,1)$	$(1,-1)$

Statement: "This game has a Nash equilibrium in pure strategies."

(b) **[3.5 points] Statement:** "The following extensive form has a proper subgame, i.e. a subgame other than the whole game."

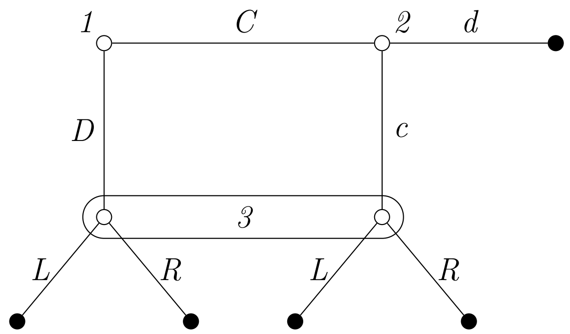

Figure 1: Extensive-form for statement (b).

Part II — Exercises

Question 3 (7 points)**.** Consider the following strategic-form game with two players who have expected utility preferences:

	L	M	R
A	$(5,2)$	$(1,0)$	$(2,0)$
B	$(0,1)$	$(3,4)$	$(2,0)$
C	$(2,3)$	$(1,3)$	$(1,0)$

(a) Perform the Cardinal Iterated Deletion of Strictly Dominated Strategies.

(b) Find all Nash equilibria, both in pure and in mixed strategies of the game.

Question 4 (7 points)**.** An entrant, Ann, first decides whether to stay *Out* of a market or to go *In*. If she stays *Out*, the incumbent, Bob, keeps the market alone and Ann earns 2 while Bob earns 5 of profits. If Ann goes *In*, the two firms must *simultaneously* commit to a technological standard — each picks the format its product will use (say, the charging cable of the product), and they sell only if their choices are compatible. The resulting profits for Ann and Bob are:

	b_1	b_2
a_1	$(4, 1)$	$(0, 0)$
a_2	$(0, 0)$	$(1, 4)$

Table 1: Simultaneous move game of technological standards.

For both firms, payoffs equal profits.

(a) Represent the situation as an extensive-form game with imperfect information. How many subgames does it have?

(b) Find all Nash equilibria, pure and mixed, of the simultaneous game in Table 1.

(c) Find all subgame-perfect equilibria of the game.

(d) Exhibit a Nash equilibrium that is *not* subgame-perfect, and explain why it fails subgame perfection.

Part III — Advanced Question

Question 5 (6 points)**.** Two flatmates simultaneously decide whether to contribute to a shared public good — for example, cleaning their common kitchen. Each player $i \in \{1, 2\}$ chooses $g_i \in \{0, 1\}$: either *Contribute* ($g_i = 1$) or *Not* ($g_i = 0$). Each contribution costs the contributor $c > 0$ and produces a benefit $b > 0$ enjoyed by *both* flatmates. Player i's payoff is

$$\pi_i(g_1, g_2) = b\,(g_1 + g_2) - c\,g_i.$$

Throughout, assume $b < c < 2b$.

(a) Interpret the assumption $b < c < 2b$ in terms of the incentives to contribute.

(b) Write the 2×2 payoff matrix of this game in terms of b and c.

(c) Find all the Nash equilibria of the game.

(d) Is any of the Nash equilibria Pareto efficient? Identify the outcome that both players prefer, and explain the conflict between individual incentives and collective welfare.

(e) One flatmate's parent, wanting to incentivise good behaviour, rewards every contribution made in the flat — by either flatmate — lowering the effective cost from c to $c - s$ (with $0 \leq s \leq c$). What is the smallest reward s that makes *Contribute* a dominant strategy for each player?

Solutions

Solution to Question 1. A finite extensive form (extensive-form game-frame) with perfect information consists of:

- a finite rooted directed tree;
- a set of players $I = \{1, \ldots, n\}$ and a function assigning one player to every decision node;
- a set of actions A and a function assigning one action to every directed edge, with no two edges out of the same node assigned the same action;
- a set of outcomes O and a function assigning one outcome to every terminal node.

Perfect information means that every player, when called to move, knows exactly which node has been reached, i.e. every information set is a *singleton*: there are no non-trivial information sets, equivalently no simultaneous moves and no hidden past choices.

What to add to obtain an extensive-form game. Preferences over outcomes.

Solution to Question 2.

(a) **False.** This is Matching Pennies. Checking best responses, no pure profile is a mutual best response: against H the row player prefers H but then the column player prefers T; against T the row player prefers T but then the column player prefers H, and so on cyclically. Hence there is *no* Nash equilibrium in pure strategies.

(b) **False.** In *Selten's horse* the whole game is the *only* subgame; there is no proper subgame. Recall that a subgame must start at a singleton node and may not cut through an information set. The only non-root singleton decision node is Player 2's node, but the subtree starting there contains only *one* of the two nodes of Player 3's information set (the node reached after c), while the other node (reached after D) lies outside that subtree. Starting a subgame at Player 2's node would therefore split Player 3's information set, which is not allowed. Player 3's two nodes are not singletons, so no subgame starts there either. Hence the whole game is the unique subgame and the statement is false.

Solution to Question 3.

(a) *Player 1.* Strategy C is strictly dominated by the mixed strategy

$$\sigma_1 = \begin{pmatrix} A & B & C \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix},$$

since against L, M, R this mixture gives Player 1 the payoffs $\frac{5}{2}, 2, 2$, while C gives $2, 1, 1$. Eliminate C.

Player 2. In the reduced game, strategy R is strictly dominated by L. Eliminate R.

The surviving game is the 2×2 game

	L	M
A	$(5,2)$	$(1,0)$
B	$(0,1)$	$(3,4)$

(b) Since strictly dominated strategies are never played in equilibrium, the Nash equilibria of the original game coincide with those of the reduced 2×2 game.

Pure equilibria: (A, L) and (B, M) (each is a mutual best response).

Mixed equilibrium: let Player 1 play A with probability p and Player 2 play L with probability q. Player 1 is indifferent when the payoffs to A and B coincide, and likewise for Player 2:

$$\underbrace{5q + 1(1-q)}_{A} = \underbrace{0\,q + 3(1-q)}_{B} \iff 4q + 1 = 3 - 3q \iff q = \tfrac{2}{7},$$

$$\underbrace{2p + 1(1-p)}_{L} = \underbrace{0\,p + 4(1-p)}_{M} \iff p + 1 = 4 - 4p \iff p = \tfrac{3}{5}.$$

Hence the third Nash equilibrium is

$$\sigma_1 = \begin{pmatrix} A & B & C \\ \frac{3}{5} & \frac{2}{5} & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} L & M & R \\ \frac{2}{7} & \frac{5}{7} & 0 \end{pmatrix}.$$

In total the game has three Nash equilibria: two in pure strategies and one in (properly) mixed strategies.

Solution to Question 4.

(a) At the root Ann chooses *Out* (terminal payoff $(2, 5)$) or *In*. After *In* the two firms move simultaneously: this is modelled by one firm choosing its action at a node and the other choosing at an information set that does not reveal the first firm's choice. The node reached immediately after *In* is a singleton and the entire simultaneous game lies below it, so it starts a proper subgame. Hence the game has **two subgames**: the whole game and the simultaneous-move game after *In* (the unique proper subgame).

(b) The subgame is a Battle-of-the-Sexes game. *Pure equilibria:* (a_1, b_1) with payoff $(4, 1)$ and (a_2, b_2) with payoff $(1, 4)$. *Mixed equilibrium:* let Ann play a_1 with probability p and Bob play b_1 with probability q. Indifference gives

$$4q = 1 - q \iff q = \tfrac{1}{5}, \qquad p = 4(1-p) \iff p = \tfrac{4}{5}.$$

So the mixed NE is $\big(a_1 : \tfrac{4}{5},\, a_2 : \tfrac{1}{5}\big)$, $\big(b_1 : \tfrac{1}{5},\, b_2 : \tfrac{4}{5}\big)$, which gives each player expected payoff $\tfrac{4}{5}$.

(c) Replace the subgame by each of its equilibrium payoffs and let Ann choose optimally at the root, comparing with her *Out* payoff of 2:

- Continuation (a_1, b_1): Ann's continuation payoff $4 > 2$, so she plays *In*. SPE $(In;\, a_1, b_1)$, outcome $(4, 1)$.
- Continuation (a_2, b_2): Ann's continuation payoff $1 < 2$, so she plays *Out*. SPE $(Out;\, a_2, b_2)$, outcome $(2, 5)$.
- Mixed continuation: Ann's continuation payoff $\tfrac{4}{5} < 2$, so she plays *Out*. SPE $(Out;\, \text{mixed})$, outcome $(2, 5)$.

There are thus three subgame-perfect equilibria; entry occurs only when the subgame is expected to coordinate on Ann's preferred standard (a_1, b_1).

(d) Consider $(Out;\, a_1, b_2)$: Ann stays *Out*, with the off-path plan a_1 for Ann and b_2 for Bob. This is a Nash equilibrium of the whole game: since *Out* is played the subgame is never reached, so Bob is indifferent across his subgame actions, and if Ann deviated to *In* the outcome (a_1, b_2) would give her $0 < 2$, so *Out* is optimal. Yet (a_1, b_2) is *not* a Nash equilibrium of the subgame (given a_1, Bob strictly prefers b_1; given b_2, Ann strictly prefers a_2). Hence the profile is not subgame-perfect: it rests on non-credible play in the subgame that is never tested on the equilibrium path.

Solution to Question 5. (For brevity write C for *Contribute* ($g_i = 1$) and N for *Not* ($g_i = 0$).)

(a) The benefit of a single contribution is b for each flatmate, while its cost c falls entirely on the contributor. So, $c > b$ means a contribution does *not* pay for the contributor alone (the private benefit b is below the cost c), so individually nobody wants to contribute. Instead, $c < 2b$ means a contribution does pay *socially*: it generates total benefit $2b$ (one b for each flatmate), which exceeds its cost c. So the assumption captures exactly the tension of a public good — privately too costly, but socially worthwhile.

(b) Using $\pi_i = b(g_1 + g_2) - c\, g_i$:

	C	N
C	$(2b - c,\ 2b - c)$	$(b - c,\ b)$
N	$(b,\ b - c)$	$(0,\ 0)$

(c) For either player, switching from N to C changes own payoff by $b - c < 0$ regardless of the other's choice:

$$(2b - c) - b = b - c < 0 \quad \text{(if the other contributes)},$$
$$(b - c) - 0 = b - c < 0 \quad \text{(if the other does not)}.$$

Hence N (free-riding) is a *strictly dominant strategy* for both players, and the unique Nash equilibrium is (N, N), with payoffs $(0, 0)$.

(d) **No.** The equilibrium (N, N) is Pareto dominated by (C, C), which gives each player $2b - c > 0$. Both players strictly prefer mutual contribution to the equilibrium, yet each individually prefers to free-ride because a single contribution returns only b to the contributor while costing $c > b$. The positive externality b that a contribution confers on the *other* flatmate is not internalized: this is the free-rider problem, and it is exactly a Prisoner's-Dilemma structure.

(e) With effective cost $c - s$, the gain from switching to C becomes $b - (c - s) = b - c + s$. *Contribute* is (weakly) dominant once this is nonnegative, i.e. $s \geq c - b$, and strictly dominant for $s > c - b$. The smallest such reward is

$$s^* = c - b.$$

A reward of $s^* = c - b$ brings the private cost down to the private benefit, so that the parent's incentive exactly offsets the temptation to free-ride. Since the reward is paid for *any* contribution in the flat, it lowers the cost symmetrically for both flatmates: each now finds it (weakly) dominant to contribute, and the efficient outcome (C, C) is restored.

Game Theory — Exam - A

Enrico Mattia Salonia

1 July 2026

Time: 75 minutes. Be clear and concise.

Part I — Definitions and True/False

Question 1 (4 points)**.** Define a **game-frame in strategic form with lotteries**. What do you need to add to get a **strategic-form game with cardinal payoffs**?

Question 2 (7 points)**.** Say whether the following statements are true, false, or uncertain, and *justify* your answer. ("uncertain" means that the statement is true under conditions that are not explicitly mentioned.)

(a) **[3.5 points]** Consider three basic outcomes o_1, o_2, o_3. A decision maker faces the *simple* lottery

$$L = \begin{pmatrix} o_1 & o_2 & o_3 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

and the *compound* lottery

$$C = \begin{pmatrix} o_1 & \hat{L} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}, \qquad \hat{L} = \begin{pmatrix} o_2 & o_3 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

Statement: "An expected utility maximizer might prefer the simple lottery L to the compound lottery C."

(b) **[3.5 points] Statement:** "The following extensive-form game has perfect recall."

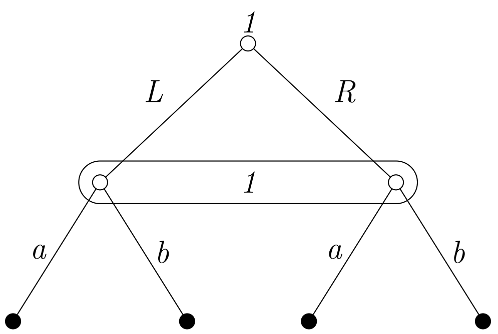

Figure 1: Extensive-form game for statement (b).

Part II — Exercises

Question 3 (7 points)**.** Consider the following strategic-form game with two players who have expected utility preferences:

	L	M	R
A	$(6,3)$	$(2,1)$	$(2,0)$
B	$(1,2)$	$(4,5)$	$(2,0)$
C	$(3,3)$	$(2,3)$	$(1,0)$

(a) Perform the Cardinal Iterated Deletion of Strictly Dominated Strategies.

(b) Find all Nash equilibria, both in pure and in mixed strategies of the game.

Question 4 (7 points)**.** Consider the following trust game. Ann has 10 euros and moves first:

- if she chooses **Keep** (K), she keeps her 10 euros and Bob gets nothing;
- if she chooses **Send** (S), she gives away all 10 euros and Bob receives 40 euros (the amount sent is quadrupled).

After S, Bob chooses a return $b \in \{0, 10, 20, 30, 40\}$ to Ann, so that the monetary rewards are $m_A = b$ and $m_B = 40 - b$. Bob's preferences are determined by Nature *before* the game begins but are *not observed* by Ann. With probability $p \in (0, 1)$ Bob is **generous**: a Beckerian altruist with utility $U_B = m_B + 2\,m_A$. With probability $1 - p$ Bob is **selfish**: he maximizes m_B only. Ann is selfish and maximizes her *expected* monetary payoff. She knows p but not Bob's realized preferences.

(a) Find the optimal return b^* for each of Bob's preference types after S.

(b) Compute Ann's expected payoff from S and from K as a function of p.

(c) Find the threshold p^* such that Ann chooses S if and only if $p > p^*$.

(d) Briefly interpret p^*: what must Ann believe about the population of potential Bobs for trust to emerge?

Part III — Advanced Question

Question 5 (6 points)**.** Two hunters simultaneously decide whether to hunt a *Stag* (S) or to hunt a *Hare* (H). Catching the stag requires the cooperation of *both* hunters: if both choose S each gets a; if a hunter chooses S while the other chooses H, the lone stag hunter catches nothing and gets 0, while the hare hunter still gets d; a hunter choosing H always catches a hare and gets d, regardless of the other. Throughout, assume $a > d > 0$.

(a) Write the 2×2 payoff matrix of this game in terms of a and d.

(b) Interpret the assumption $a > d > 0$ in terms of the incentives to hunt the stag.

(c) Find all the Nash equilibria of the game in pure strategies.

(d) Find the Nash equilibrium in mixed strategies, and compute the probability with which each hunter chooses the stag.

(e) Is any of the Nash equilibria Pareto efficient? Explain why the inefficient equilibrium can nonetheless be played.

Solutions

Solution to Question 1. A *game-frame in strategic form with lotteries* is a quadruple

$$\langle I, (S_i)_{i\in I}, O, f\rangle$$

where:

- $I = \{1, \ldots, n\}$ is the set of players ($n \geq 2$);
- for each player $i \in I$, S_i is the set of strategies, with $S = S_1 \times \cdots \times S_n$ the set of strategy profiles;
- O is a set of basic outcomes;
- $f : S \to \mathcal{L}(O)$ is a probabilistic outcome function assigning to each strategy profile a lottery over O.

The only difference from an ordinal game-frame is that a strategy profile now induces a *lottery* over basic outcomes rather than a single deterministic outcome.

What to add to obtain a strategic-form game with cardinal payoffs. For each player i, a von Neumann–Morgenstern ranking $\succsim_i$ of the lotteries in $\mathcal{L}(O)$, turning the frame into the quintuple $\langle I, (S_i)_{i\in I}, O, f, (\succsim_i)_{i\in I}\rangle$. These expected-utility preferences are what allow the players to evaluate the lotteries induced by f.

Solution to Question 2.

(a) **False.** Reduce the compound lottery C to its associated simple lottery by computing, for each basic outcome, the total probability with which it is realized through all branches of C:

$$o_1 : \tfrac{1}{2}, \qquad o_2 : \tfrac{1}{2}\cdot\tfrac{1}{2} = \tfrac{1}{4}, \qquad o_3 : \tfrac{1}{2}\cdot\tfrac{1}{2} = \tfrac{1}{4}.$$

Hence C reduces to $\begin{pmatrix} o_1 & o_2 & o_3 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4}\end{pmatrix}$, which is exactly L. By the reduction of compound lotteries (an expected utility maximizer cares only about the induced probability distribution over basic outcomes), L and C yield the same expected utility, so the agent is *indifferent* between them and can never strictly prefer one to the other. The statement is therefore false.

(b) **False.** Perfect recall requires that a player never forgets her own past actions: if two nodes lie in the same information set, they must be reached by the same sequence of that player's own past choices. Here Player 1 moves at the root (choosing L or R) and then again at the information set $\{nL, nR\}$. But nL is reached after her own action L and nR after her own action R; grouping them in a single information set means that at her second move Player 1 cannot tell which action she herself just took. This violates perfect recall, so the statement is false.

Solution to Question 3.

(a) *Player 1.* Note first that no *pure* strategy strictly dominates C: the pure strategy $A = (6, 2, 2)$ only *weakly* dominates $C = (3, 2, 1)$, since A ties C against M (both give 2) even though it does strictly better against L and R; and B does not dominate C at all (against L, B gives $1 < 3$). However, C *is* strictly dominated by the mixed strategy

$$\sigma_1 = \begin{pmatrix} A & B & C \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix},$$

since against L, M, R this mixture gives Player 1 the payoffs $\frac{7}{2}, 3, 2$, strictly above C's $3, 2, 1$ in every column. Because Cardinal IDSDS allows deletion by mixed strategies, eliminate C.

Player 2. In the reduced game, strategy R is strictly dominated by L (against A, $3 > 0$; against B, $2 > 0$). Eliminate R.

The surviving game is the 2×2 coordination (Battle-of-the-Sexes) game

	L	M
A	$(6,3)$	$(2,1)$
B	$(1,2)$	$(4,5)$

(b) Since strictly dominated strategies are never played in equilibrium, the Nash equilibria of the original game coincide with those of the reduced 2×2 game.

Pure equilibria: (A, L) and (B, M) (each is a mutual best response).

Mixed equilibrium: let Player 1 play A with probability p and Player 2 play L with probability q. Indifference gives

$$\underbrace{6q + 2(1-q)}_{A} = \underbrace{1\,q + 4(1-q)}_{B} \iff 4q + 2 = 4 - 3q \iff q = \tfrac{2}{7},$$

$$\underbrace{3p + 2(1-p)}_{L} = \underbrace{1\,p + 5(1-p)}_{M} \iff p + 2 = 5 - 4p \iff p = \tfrac{3}{5}.$$

Hence the third Nash equilibrium is

$$\sigma_1 = \begin{pmatrix} A & B & C \\ \frac{3}{5} & \frac{2}{5} & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} L & M & R \\ \frac{2}{7} & \frac{5}{7} & 0 \end{pmatrix}.$$

In total the game has three Nash equilibria: two in pure strategies and one in (properly) mixed strategies.

Solution to Question 4.

(a) Preference-contingent optimal return after S, recalling $m_A = b$ and $m_B = 40 - b$:

- **Generous type** ($U_B = m_B + 2m_A$):

  $$U_B = (40 - b) + 2b = 40 + b,$$

  which is increasing in b, hence $b_G^* = 40$.

- **Selfish type** ($U_B = m_B$):

  $$U_B = 40 - b,$$

  which is decreasing in b, hence $b_S^* = 0$.

(b) Anticipating $b_G^* = 40$ and $b_S^* = 0$, Ann's expected payoff from sending is

$$\mathbb{E}[m_A \mid S] = p \cdot 40 + (1-p) \cdot 0 = 40p, \qquad m_A(K) = 10.$$

(c) Ann chooses S iff

$$40p > 10 \iff p > \tfrac{1}{4}.$$

Therefore the threshold is

$$p^* = \tfrac{1}{4}.$$

At $p = \frac{1}{4}$, Ann is indifferent between K and S.

(d) Trust emerges only if Ann believes the probability of meeting a generous Bob is sufficiently high: here she must believe that at least one quarter of potential partners are generous. Below that threshold the risk of meeting a selfish Bob (who returns nothing) outweighs the gain from a generous one, and Ann prefers to keep her money.

Solution to Question 5.

(a) With rows for the first hunter and columns for the second, the payoff matrix is

	S	H
S	(a, a)	$(0, d)$
H	$(d, 0)$	(d, d)

Both choosing S gives (a, a); a lone stag hunter gets 0 while the partner who took the hare gets d; both choosing H gives (d, d).

(b) Hunting the stag yields the highest payoff a, but only if the other hunter cooperates; a lone stag hunter gets 0. Hunting the hare guarantees $d > 0$ regardless of the other. So $a > d$ means cooperation on the stag is the most rewarding outcome, while $d > 0$ means the hare is a *safe* option that pays a positive amount even if the partner does not cooperate. The assumption captures the tension between a rewarding but risky joint action (stag) and a safe individual action (hare).

(c) Check mutual best responses. (S, S): given the other plays S, $a > d$ so S is a best response for each; hence (S, S) is a Nash equilibrium with payoff (a, a). (H, H): given the other plays H, $d > 0$ so H is a best response for each; hence (H, H) is a Nash equilibrium with payoff (d, d). So the two pure Nash equilibria are (S, S) and (H, H).

(d) Let each hunter play S with probability p. The opponent is indifferent between S and H when

$$\underbrace{a\,p + 0\,(1-p)}_{S} = \underbrace{d\,p + d\,(1-p)}_{H} \iff a\,p = d \iff p = \frac{d}{a}.$$

By symmetry both hunters choose the stag with probability $p^* = d/a \in (0, 1)$, which is the mixed Nash equilibrium.

(e) **Only** (S, S) **is Pareto efficient**: it gives (a, a), which Pareto dominates (d, d) since $a > d$. Nevertheless (H, H) is also a Nash equilibrium and can be played: if each hunter believes the other will not cooperate, hunting the hare is a best response, and these pessimistic beliefs are self-fulfilling. Because the stag requires coordination and the lone stag hunter risks getting 0, *strategic uncertainty* about the partner's choice can trap the hunters in the inefficient but safe equilibrium.

Game Theory — Exam

Enrico Mattia Salonia

September 2026

Time: 75 minutes. Be clear and concise.

Part I — Definitions and True/False

Question 1 (4 points)**.** Give the definition of *strategy* for a game with imperfect information.

Question 2 (7 points)**.** Say whether the following statements are true, false, or uncertain, and *justify* your answer. ("uncertain" means that the statement is true under conditions that are not explicitly mentioned.)

(a) **[3.5 points]** Consider the following strategic-form game:

	L	R
U	$(2,1)$	$(0,0)$
D	$(1,0)$	$(1,2)$

Statement: "Because neither player has a strictly dominant strategy, this game has no Nash equilibrium in pure strategies."

(b) **[3.5 points]** Consider the following extensive-form game with perfect information:

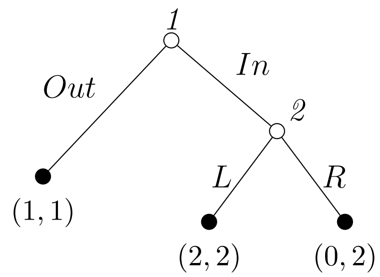

Figure 1: Extensive-form game for statement (b).

Statement: "Backward induction selects a unique strategy profile in this game."

Part II — Exercises

Question 3 (7 points)**.** Consider the following strategic-form game with two players who have expected utility preferences:

	L	M	R
A	$(1,3)$	$(1,0)$	$(2,3)$
B	$(1,0)$	$(2,0)$	$(5,2)$
C	$(4,4)$	$(2,0)$	$(0,1)$

(a) Perform the Cardinal Iterated Deletion of Strictly Dominated Strategies.

(b) Find all Nash equilibria, both in pure and in mixed strategies.

Question 4 (7 points)**.** Ann and Bob are hunters. Ann first decides whether to *Stay home* or to *Set out* on a hunting expedition. If she stays home, both spend the day on their usual work and receive payoffs $(3, 3)$. If she sets out, Ann and Bob separate in the forest and each chooses *simultaneously* whether to hunt a stag (S) or a hare (H). A stag can be caught only if both hunters pursue it, whereas a hunter can catch a hare alone. Their payoffs are:

	S	H
S	$(5, 5)$	$(0, 2)$
H	$(2, 0)$	$(2, 2)$

The first payoff is Ann's and the second is Bob's. Throughout this question, restrict attention to pure strategies.

(a) Represent the full situation as an extensive-form game with imperfect information. How many subgames does it have?

(b) Find all pure-strategy Nash equilibria of the simultaneous game following *Set out*.

(c) Find all pure-strategy subgame-perfect equilibria of the full game.

(d) Give a Nash equilibrium of the full game that is not subgame-perfect, and explain why it fails subgame perfection.

Part III — Advanced Question

Question 5 (6 points)**.** Ann runs a small online shop and hires Bob to prepare an advertising campaign. Once he begins the job, Bob decides whether to **work hard** (H) or to **take it easy** (L). Ann works from another city, so she cannot watch how much effort Bob puts into the campaign. At the end, however, she observes whether the campaign succeeds or fails. Thus, Ann sees the result of Bob's work, but not his effort.

Working hard costs Bob 2 payoff points, while taking it easy costs him nothing. If he works hard, the probability that the campaign succeeds is $\frac{4}{5}$. If he takes it easy, the probability of success is $\frac{2}{5}$. A successful campaign earns Ann 15; an unsuccessful campaign earns her nothing.

Ann can promise Bob a bonus $b \geq 0$ if the campaign succeeds; he receives no bonus if it fails. Bob's payoff is the bonus he receives minus his effort cost. Ann's payoff is the campaign's earnings minus the bonus. If Bob is indifferent between working hard and taking it easy, assume that he works hard.

(a) Calculate Bob's expected payoff if he works hard and if he takes it easy, as a function of the bonus b.

(b) Find the smallest bonus that makes working hard at least as attractive to Bob as taking it easy.

(c) Using this bonus, calculate Ann's expected payoff if Bob works hard.

(d) Suppose Ann offers no bonus. Which effort does Bob choose, and what is Ann's expected payoff? Does Ann prefer to offer the bonus found in part (b)?

(e) Explain briefly why a bonus for success can encourage Bob to work hard even though Ann cannot watch him. If the campaign succeeds, can Ann be certain that Bob worked hard?

Solutions

Solution to Question 1. A strategy for a player in a game with imperfect information is a list of choices, one for each information set of that player. It is a complete contingent plan: it specifies what the player would choose at every information set at which she might be called to move, including information sets that are not reached during play.

Solution to Question 2.

(a) **False.** A Nash equilibrium does not require dominant strategies. The game has two pure-strategy Nash equilibria. At (U, L), U is a best response to L and L is a best response to U. At (D, R), D is a best response to R and R is a best response to D.

(b) **False.** At Player 2's decision node, both L and R give Player 2 a payoff of 2. Therefore both choices are optimal. If Player 2 chooses L, Player 1 prefers In, since $2 > 1$. This gives the strategy profile (In, L). If Player 2 chooses R, Player 1 prefers Out, since $1 > 0$. This gives the strategy profile (Out, R). Backward induction therefore produces two subgame-perfect equilibria because Player 2 is indifferent.

Solution to Question 3. For Player 2, M is strictly dominated by R, since Player 2's payoff from R is strictly positive against every row while the payoff from M is always zero. For Player 1, A is strictly dominated by the mixed strategy that puts probability 1/2 on B and 1/2 on C. This mixture gives Player 1 the payoff vector

$$\tfrac{1}{2}(1, 2, 5) + \tfrac{1}{2}(4, 2, 0) = (2.5, 2, 2.5),$$

which is strictly larger than A's vector $(1, 1, 2)$. The reduced game is

	L	R
B	$(1, 0)$	$(5, 2)$
C	$(4, 4)$	$(0, 1)$.

Its pure Nash equilibria are (B, R) and (C, L).

For the mixed equilibrium, let p be the probability that Player 1 plays B, and let q be the probability that Player 2 plays R. Player 2 is indifferent when

$$2p + 1(1 - p) = 0p + 4(1 - p),$$

which gives $p = 3/5$. Player 1 is indifferent when

$$5q + 1(1 - q) = 0q + 4(1 - q),$$

which gives $q = 3/8$. Hence the mixed equilibrium is

$$\sigma_1 = (0,\ 3/5,\ 2/5), \qquad \sigma_2 = (5/8,\ 0,\ 3/8),$$

where the entries follow the displayed orders (A, B, C) and (L, M, R).

Solution to Question 4.

(a) At the root Ann chooses *Stay home*, which gives $(3, 3)$, or *Set out*. After *Set out*, the simultaneous choice can be represented by letting Ann move first in the tree and placing Bob's two decision nodes in one information set, so that Bob does not observe Ann's choice. The game has two subgames: the whole game and the simultaneous-move game following *Set out*.

(b) The simultaneous game has two pure Nash equilibria: (S, S), with payoff $(5, 5)$, and (H, H), with payoff $(2, 2)$.

(c) Compare Ann's payoff from staying home, 3, with her payoff under each equilibrium of the continuation game. With continuation (S, S), she obtains $5 > 3$, so she sets out. With continuation (H, H), she obtains $2 < 3$, so she stays home. Thus there are two subgame-perfect equilibria: one with *Set out* followed by (S, S), and the other with *Stay home* and continuation (H, H).

(d) Consider the profile in which Ann stays home but plans to hunt stag after *Set out*, while Bob plans to hunt hare. This is a Nash equilibrium of the full game: if Ann deviates to *Set out*, her best possible choice against H gives her only 2, below her payoff of 3 from staying home; Bob's action is irrelevant when Ann stays home. However, (S, H) is not a Nash equilibrium of the continuation game, since Ann would switch to H. The profile is therefore not subgame-perfect.

Solution to Question 5.

(a) Bob's expected payoffs are

$$U_B(H) = \frac{4}{5}b - 2, \qquad U_B(L) = \frac{2}{5}b.$$

(b) Working hard is at least as attractive as taking it easy when

$$\frac{4}{5}b - 2 \geq \frac{2}{5}b.$$

Therefore $\frac{2}{5}b \geq 2$, or $b \geq 5$. The smallest bonus is $b = 5$. At this bonus Bob is indifferent, so he works hard by the assumption in the question.

(c) With $b = 5$, Ann receives $15 - 5 = 10$ after success and 0 after failure. Her expected payoff when Bob works hard is

$$\frac{4}{5}(10) + \frac{1}{5}(0) = 8.$$

(d) With no bonus, working hard gives Bob -2, while taking it easy gives him 0. He therefore takes it easy. Ann's expected payoff is

$$\frac{2}{5}(15) + \frac{3}{5}(0) = 6.$$

Since $8 > 6$, Ann prefers to offer the bonus $b = 5$.

(e) Ann cannot base Bob's payment directly on effort because she cannot watch him. Success is more likely when he works hard, so the bonus makes working hard more attractive. A successful campaign does not prove that he worked hard: the campaign can still succeed when he takes it easy.