Microeconomics I — Exam

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Time: 120 minutes.

Part I — Definitions and Concepts

Question 1 (4 points)**.** Describe the primitive components of the von Neumann and Morgenstern framework for choice under risk. Explain what a compound lottery is. Write the functional form of the expected utility representation.

Question 2 (5 points)**.** Formally state the von Neumann–Morgenstern axioms. Identify which axiom you consider the most problematic, and provide a brief justification for your choice.

Question 3 (4 points)**.** Define a pure exchange economy, specifying all necessary components: the set of individuals, the consumption space, initial endowments, preferences, and feasible allocations. Define an allocation rule. Explain the conceptual difference between the budget set in individual consumer theory and the budget set in an exchange economy.

Question 4 (5 points)**.** Define a Pareto Optimal allocation and allocations satisfying the no-envy criterion. Which allocation rules implement Pareto optimal allocations, and which implement envy-free allocations? You do not need to define these rules formally, simply name them and briefly explain their underlying logic. Feel free to refer to results we covered in class.

Part II — Exercises

Question 5 (4 points)**.** Consider an insurance problem in a framework with uncertain states. There are two states of the world. An individual has a wealth of w, but accidents may occur. In state 1 ("no accident"), occurring with probability $p_1 > 0$, income is w. In state 2 ("accident"), occurring with probability $p_2 > 0$, there is a loss $D > 0$, so income is $w - D$. A *contingent consumption plan* is a pair (x_1, x_2), where x_s is consumption in state s.

An insurance contract specifies transfers in each state (z_1, z_2): the individual pays $-z_1$ in state 1 (a premium, so $z_1 < 0$) and receives z_2 in state 2 (a payout, so $z_2 > 0$). Assume the insurance company makes zero expected profit:

$$p_1 z_1 + p_2 z_2 = 0.$$

The contract (z_1, z_2) therefore yields the contingent consumption plan

$$(x_1, x_2) = (w + z_1,\ w - D + z_2).$$

1. Draw a graph with x_1 on the horizontal axis and x_2 on the vertical axis. Add the 45-degree line $x_1 = x_2$. What does this line represent in economic terms?

2. What is the contingent consumption plan if the individual buys no insurance? Show the point in the graph.

3. Draw the budget line representing all achievable consumption plans (x_1, x_2). Use $x_1 = w + z_1$ and $x_2 = w - D + z_2$ to rewrite the zero-profit condition $p_1 z_1 + p_2 z_2 = 0$ as an equation directly in (x_1, x_2). What is the slope of the budget line?

4. Assume the individual has expected utility $U(x_1, x_2) = p_1 u(x_1) + p_2 u(x_2)$ with u increasing and strictly concave. What is the individual attitude towards risk? Sketch two indifference curves: one through the no-insurance point and another one through a point in which the individual is fully insured.

Question 6 (4 points)**.** Consider an exchange economy with a single individual, Robinson Crusoe (R), and two goods. The initial endowment is $e = (e_1, e_2)$, with $e_1 > 0$ and $e_2 > 0$ (subscripts refer to goods, since there is only one individual). Robinson Crusoe's preferences are represented by the Cobb–Douglas utility function

$$u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}, \qquad \alpha \in (0, 1).$$

Show that there exist prices $p = (p_1, p_2)$ such that (p, e) is a competitive equilibrium allocation. You may proceed in **either** of the following ways:

(A) Compute the Walrasian demand $D_R(p, e)$ and find a condition on prices such that $D_R(p, e) = e$.

(B) Use a mathematical theorem covered in class to show that such prices exist. You do not need to be fully rigorous, just clearly state the theorem you are using and explain how its main hypotheses are satisfied in this case.

Part III — Advanced Questions (choose one)

Answer only **one** of Questions 7 or 8.

Question 7 (5 points)**.** In this question, you will study a framework for choice under uncertainty introduced by Luce and Raiffa (1957), which is distinct from both von Neumann–Morgenstern and Savage. The framework consists of the following components:

- A finite set of **actions** A. For example, $a \in A$ might represent "go out" or "stay home".
- A finite set of **states** of nature S. For example, $s \in S$ might represent "it rains" or "it does not rain".
- A finite set of **consequences** X. For example, $x \in X$ might represent "get wet" or "stay dry".
- A consequence function $c : A \times S \to X$ that assigns a consequence to each action–state pair. For instance, if action a is "go out" and state s is "it rains", then $c(a, s) = x$ is "get wet".

The individual has a preference relation $\succsim$ over the set of actions A, where $a \succsim a'$ means that action a is weakly preferred to action a'. Answer the following:

1. Is this framework closer to von Neumann–Morgenstern or Savage in how it represents uncertainty?

2. Here it is possible for an individual to strictly prefer an action to another regardless of their consequences. For example, $a \succ a'$ could hold even if $c(a, s) = c(a', s)$ for all s. Discuss whether this is permitted in the von Neumann–Morgenstern expected utility framework.

3. Propose a formal axiom on the preference relation $\succsim$ over A stating that the individual ranks actions solely based on their consequences. That is, if two actions yield the same consequences in every state, the individual must be indifferent between them.

4. Assume the individual satisfies the axiom proposed in point 3. What expected utility representation of $\succsim$ over A could be obtained?

Question 8 (5 points)**.** In this question you will study a fairness concept for allocations in exchange economies that we did not cover in class, introduced by Pazner and Schmeidler (1978): *egalitarian equivalence*. Consider a pure exchange economy with a finite set of individuals. Let $\overline{e} = (\overline{e}^1, \ldots, \overline{e}^L)$ be the total amount of resources in the economy. A feasible allocation x is **egalitarian equivalent** if there exists a number $\lambda > 0$ such that, for each individual i,

$$x_i \sim_i \lambda \overline{e}.$$

Answer the following:

1. Explain in words what egalitarian equivalence means. Why is it considered a fairness criterion? How does it differ from the **equal division** allocation, where $x_i = \frac{1}{n} \sum_j e_j$ for all i?

2. There is a difference behind the thought experiment inspiring egalitarian equivalence and no-envy, what is it?

3. Show that the equal division allocation is always egalitarian equivalent.

4. There is a simple graphical test to check whether an allocation is egalitarian equivalent in a two-individual, two-good economy. Consider the picture below. Is the allocation x egalitarian equivalent? Describe this test and explain how it works.

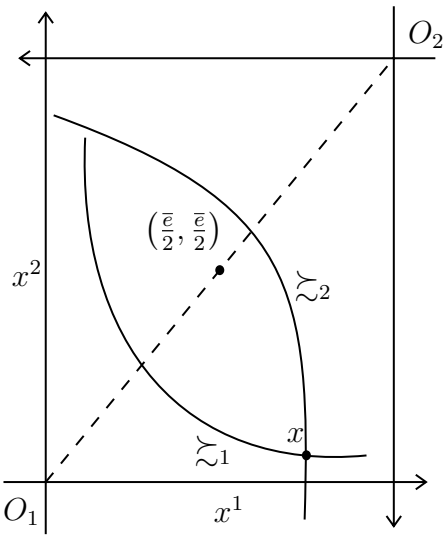


Figure 1: Exchange economy

Solutions

Solution to Question 1. The primitives are a finite set of outcomes X, the set of simple lotteries $\Delta(X) = \left\{p : X \to [0,1] : \sum_{x \in X} p(x) = 1\right\}$ and a preference relation $\succsim$ over $\Delta(X)$. A compound lottery delivers lotteries as its prizes. For example, it delivers p with probability α and q with probability $1-\alpha$. Its reduced simple lottery is $r = \alpha p + (1-\alpha)q$, where $r(x) = \alpha p(x) + (1-\alpha)q(x)$.

An expected utility representation has the form

$$U(p) = \sum_{x \in X} p(x)u(x), \qquad u : X \to \mathbb{R},$$

with $p \succsim q$ if and only if $U(p) \geq U(q)$. The function u is the von Neumann–Morgenstern utility function over outcomes. □

Solution to Question 2. For all lotteries $p, q, r \in \Delta(X)$, the axioms are:

1. **Weak order:** preferences are complete, meaning $p \succsim q$ or $q \succsim p$ (or both), and transitive, meaning
$$p \succsim q, \quad q \succsim r \quad \Longrightarrow \quad p \succsim r.$$
2. **Continuity:** if $p \succ q \succ r$, there exist $\alpha, \beta \in (0,1)$ such that
$$\alpha p + (1-\alpha)r \succ q \succ \beta p + (1-\beta)r.$$
3. **Independence:** for every $\alpha \in (0,1)$,
$$p \succsim q \quad \Longleftrightarrow \quad \alpha p + (1-\alpha)r \succsim \alpha q + (1-\alpha)r.$$

A defensible choice of the most problematic axiom is independence: adding the same consequence to both options can change an individual's ranking, particularly when it introduces or removes certainty. Such Allais-type choices conflict with independence. Other choices are acceptable if justified. □

Solution to Question 3. There is a finite set of individuals $I = \{1, \ldots, n\}$ and ℓ goods. Each individual has consumption space $\mathbb{R}^\ell_+$, an initial endowment $e_i \in \mathbb{R}^\ell_+$, and preferences $\succsim_i$ over that consumption space. With the individual and good sets fixed, an economy is the profile $E = (\succsim_i, e_i)_{i \in I}$.

An allocation is a vector $x = (x_1, \ldots, x_n)$, with $x_i \in \mathbb{R}^\ell_+$. It is feasible if

$$\sum_{i \in I} x_i = \sum_{i \in I} e_i.$$

Let $F(E)$ be the set of feasible allocations. An allocation rule assigns to each economy a subset $R(E) \subseteq F(E)$.

In individual consumer theory, income m is exogenous and the budget set is $\{x_i \geq 0 : p \cdot x_i \leq m\}$. In an exchange economy, income is the value of the endowment, so

$$B_i(p, e_i) = \{x_i \geq 0 : p \cdot x_i \leq p \cdot e_i\}.$$

Thus a price change affects both the cost of consumption and the individual's income. □

Solution to Question 4. A feasible allocation x is **Pareto optimal** if no feasible allocation y satisfies $y_i \succsim_i x_i$ for every i and $y_j \succ_j x_j$ for at least one j. It is **envy-free** if

$$x_i \succsim_i x_j \qquad \text{for all individuals } i, j.$$

Each individual evaluates both bundles using their own preferences.

Under local non-satiation, the First Welfare Theorem implies that the **Walrasian rule** and the **Walrasian rule with transfers** select Pareto optimal allocations whenever the relevant equilibria exist: consumers optimise at common prices and markets clear.

The **Egalitarian Walrasian rule** first assigns everyone an equal share of total endowments, then allows competitive trade. Everyone faces the same budget set, so no individual prefers someone else's chosen bundle to their own. Its allocations are envy-free and, under local non-satiation, Pareto optimal. Equal division itself is also envy-free, though it need not be Pareto optimal. □

Solution to Question 5. Since the two states exhaust all possibilities, $p_1 + p_2 = 1$.

1. The 45-degree line $x_1 = x_2$ represents certain consumption: the individual consumes the same amount in both states and is fully insured.

2. Without insurance, the plan is the endowment $E = (w, w - D)$, below the 45-degree line.

3. Substitution into the zero-profit condition gives

   $$p_1(x_1 - w) + p_2\big(x_2 - (w - D)\big) = 0.$$

   Hence the fair-insurance budget line is

   $$p_1 x_1 + p_2 x_2 = p_1 w + p_2(w - D) = w - p_2 D,$$

   or

   $$x_2 = \frac{w - p_2 D}{p_2} - \frac{p_1}{p_2} x_1.$$

   Its slope is $-p_1/p_2$, and it passes through E. The restrictions $z_1 \leq 0$, $z_2 \geq 0$, including no insurance as a boundary case, select the part with $x_1 \leq w$ and $x_2 \geq w - D$, within the consumption domain.

4. Strict concavity of u implies strict risk aversion. The indifference curves are decreasing and strictly convex. If u is differentiable, their slope is

   $$-\frac{p_1 u'(x_1)}{p_2 u'(x_2)}.$$

   The fair full-insurance plan is $F = (c, c)$, with $c = w - p_2 D$. It is optimal, since strict Jensen's inequality gives, for every nonconstant feasible plan,

   $$p_1 u(x_1) + p_2 u(x_2) < u(p_1 x_1 + p_2 x_2) = u(c).$$

   In particular, $U(F) > U(E)$. At F, a differentiable indifference curve is tangent to the budget line.

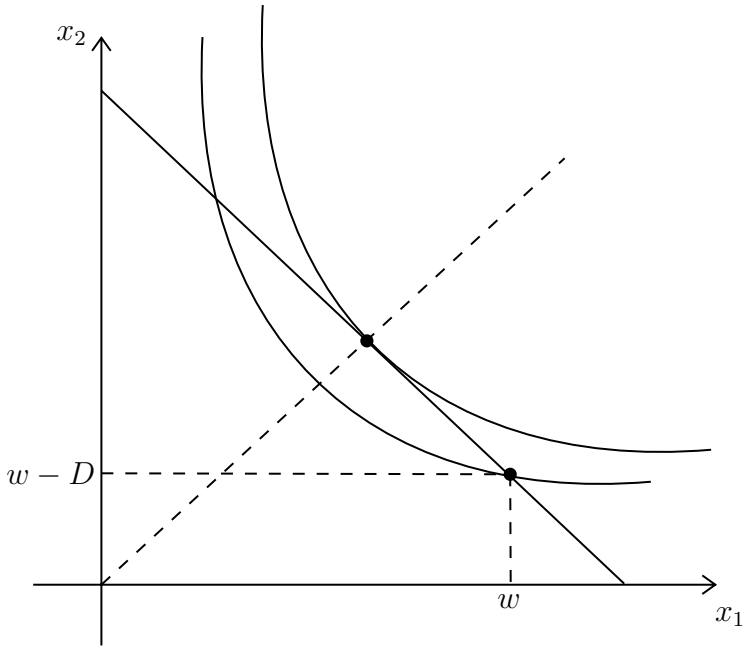


Figure 2: Insurance Problem

□

Solution to Question 6. Either argument suffices.

(A) For strictly positive prices, write $m = p_1 e_1 + p_2 e_2$. Cobb–Douglas demand is

$$D_R^1(p, e) = \alpha \frac{m}{p_1}, \qquad D_R^2(p, e) = (1 - \alpha) \frac{m}{p_2}.$$

Requiring $D_R(p, e) = e$ gives

$$(1 - \alpha) p_1 e_1 = \alpha p_2 e_2, \qquad \frac{p_1}{p_2} = \frac{\alpha e_2}{(1 - \alpha) e_1}.$$

For example, take

$$p = \left(\frac{\alpha}{e_1}, \frac{1 - \alpha}{e_2} \right) \gg 0.$$

Then $p \cdot e = 1$ and the demands equal e_1, e_2. The only consumer optimises at e, and both markets clear, so (p, e) is a competitive equilibrium.

(B) Apply the **supporting hyperplane theorem** to

$$C = \{x \in \mathbb{R}^2_+ : u(x) \geq u(e)\}.$$

This set is closed by continuity and convex because Cobb–Douglas utility is concave. Since $e \gg 0$, it lies on the boundary of C. The theorem gives a nonzero supporting normal p with

$$p \cdot x \geq p \cdot e \qquad \text{for every } x \in C.$$

At the interior bundle e, smoothness gives a supporting normal proportional to $\nabla u(e) \gg 0$, since both marginal utilities are strictly positive. Every strictly preferred bundle lies in the interior of C, so it satisfies $p \cdot x > p \cdot e$ and is unaffordable. Thus e maximises utility over the budget set. With one consumer, consuming e clears both markets, establishing equilibrium.

□

Solution to Question 7.

1. It is closer to Savage: uncertainty is described by states of nature and each action induces a state-contingent consequence function. Objective probabilities are not specified.

2. In the von Neumann–Morgenstern framework the objects of choice are lotteries over outcomes. Two labels describing the same lottery refer to the same choice object and must be indifferent. A strict preference between consequence-identical actions cannot be represented unless the outcome description is enriched to include something else the individual values about the action.

3. A suitable **consequentialism** axiom is

$$\big[c(a,s) = c(a',s) \text{ for every } s \in S\big] \quad \Longrightarrow \quad a \sim a'.$$

4. The natural candidate is **subjective expected utility**:

$$U(a) = \sum_{s \in S} \pi(s) u(c(a,s)), \qquad \pi(s) \geq 0, \quad \sum_{s \in S} \pi(s) = 1,$$

representing preferences by $a \succsim a'$ if and only if $U(a) \geq U(a')$.

□

Solution to Question 8.

1. Egalitarian equivalence requires a common reference bundle $\lambda \overline{e}$ such that each individual is indifferent between their own allocation and that reference bundle, according to their own preferences. The fairness idea is equality of the reference bundle used to assess everyone's allocation. Actual bundles may differ, and λ need not equal $1/n$. The reference bundle is hypothetical and need not be feasible for everyone to receive simultaneously. Equal division instead gives everyone exactly $\overline{e}/n$.

2. Under egalitarian equivalence, each individual compares their own bundle with a common hypothetical benchmark. Under no-envy, each individual compares their own bundle with every other individual's actual bundle. The latter asks whether anyone would prefer to exchange bundles.

3. Under equal division,

$$x_i = \frac{1}{n} \sum_j e_j = \frac{1}{n} \overline{e} \qquad \text{for every } i.$$

Setting $\lambda = 1/n > 0$ gives $x_i \sim_i \lambda \overline{e}$, so equal division is always egalitarian equivalent.

4. For the monotone preferences depicted, find the intersection of each individual's indifference curve through x with the diagonal joining the two origins. Measure the proportional distance λ_i from that individual's *own* origin: consumer 1 from O_1, and consumer 2 from O_2. Each intersection then represents $\lambda_i \overline{e}$ for the relevant consumer.

The allocation is egalitarian equivalent if and only if $\lambda_1 = \lambda_2$. In the Edgeworth box, the two intersection points must therefore be symmetric about the midpoint $(\overline{e}/2, \overline{e}/2)$; they need not coincide. Equivalently, reflect consumer 2's curve through the midpoint and check whether it and consumer 1's curve meet the diagonal at the same point.

In the supplied figure the intersections are not symmetric: consumer 2's proportional distance from O_2 is greater than consumer 1's from O_1. Thus $\lambda_2 > \lambda_1$, and x is **not** egalitarian equivalent.

□