Game Theory — Mock Exam

Enrico Mattia Salonia

Academic Year 2025–2026

Time: 75 minutes. Be clear and concise.

Part I — Definitions and True/False

Question 1 (4 points)**.** Define an ordinal game in strategic form.

Question 2 (7 points)**.** Say whether the following statements are true, false or uncertain and *justify* your answer. ('uncertain' means that the statement is true under conditions that are not explicitly mentioned in it)

(a) **[3.5 points]** Consider the following strategic-form game.

	L	R
U	$\begin{pmatrix} o_1 & o_2 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	o_3
D	o_4	$\begin{pmatrix} o_3 & o_4 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Players' preferences over outcomes are:

$$o_1 \succ_1 o_3 \succ_1 o_4 \succ_1 o_2, \qquad o_4 \succ_2 o_1 \succ_2 o_3 \succ_2 o_2.$$

Statement: "There is a Nash equilibrium".

(b) **[3.5 points] Statement:** "The following extensive form has two subgames, of which only one is a proper subgame (i.e., not the whole game)".

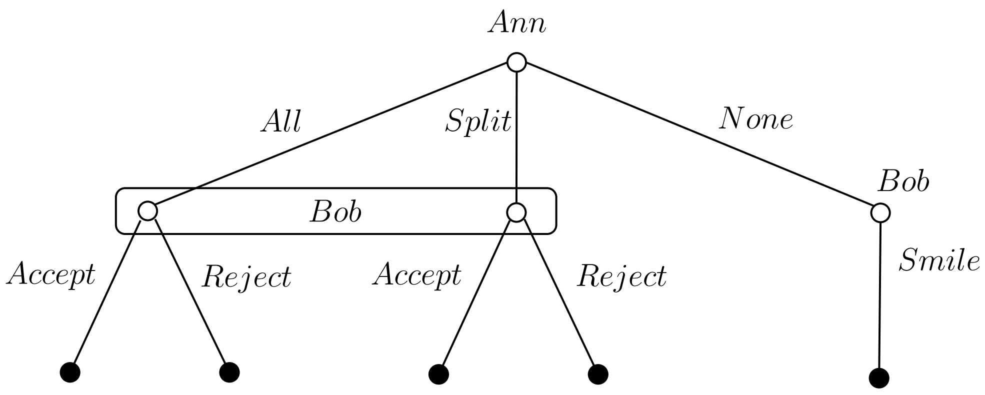

Figure 1: Extensive-form game for statement (b).

Part II — Exercises

Question 3 (7 points)**.** Consider the following strategic-form game with two players who have expected utility preferences:

	L	M	R
A	$(2,1)$	$(0,2)$	$(1,0)$
B	$(0,2)$	$(2,1)$	$(1,0)$
C	$(0,0)$	$(0,0)$	$(0,3)$

1. Perform the Cardinal Iterated Deletion of Strictly Dominated Strategies.

2. Find all Nash equilibria in mixed strategies of the game.

Question 4 (7 points)**.** Consider the following trust game. Ann has 10 euros and moves first:

- if she chooses **Keep** (K), she keeps her 10 euros and Bob gets nothing;
- if she chooses **Send** (S), Bob receives 30 euros.

After S, Bob chooses a return $b \in \{0, 10, 20, 30\}$ to Ann. Bob's preferences are determined by Nature *before* the game begins but they are *not observed* by Ann. With probability $p \in (0, 1)$ Bob is **generous**: a Beckerian altruist with utility $U_B = m_B + 2\,m_A$. With probability $1 - p$ Bob is **selfish**: he maximizes m_B only. Ann is selfish and maximizes her *expected* monetary payoff. She knows p but not Bob's realized preferences.

1. Find the optimal return b^* for each of Bob's preferences after S.
2. Compute Ann's expected payoff from S and from K as a function of p.
3. Find the threshold p^* such that Ann chooses S if and only if $p > p^*$.
4. Briefly interpret p^*: what must Ann believe about the population of potential Bobs for trust to emerge?

Part III — Advanced Question

Question 5 (6 points)**.** Two energy companies, Firm 1 and Firm 2, compete for drilling rights to a newly discovered oil field. Each firm submits one sealed bid without observing the other bid. The higher bidder wins the concession and pays exactly its own bid. If bids are equal, each firm wins with probability $\frac{1}{2}$. Both firms have expected utility preferences.

The oil field has a common economic value that can be $V = 0$ or $V = 100$. Both firms believe:

$$p(V = 100) = \tfrac{1}{2}, \qquad p(V = 0) = \tfrac{1}{2}.$$

Bids are restricted to three pure strategies $\{40, 60, 80\}$.

1. Compute each firm's expected value of the oil field.
2. Build the expected-payoff table (Firm 1 rows, Firm 2 columns).
3. Find all pure-strategy Nash equilibria.
4. At each pure-strategy equilibrium, does the winning firm pay more than the expected value of the oil field?
5. Now suppose that a regulatory constraint removes bid 40 from the feasible set, so that each firm can only bid from $\{60, 80\}$. Find the pure-strategy Nash equilibrium of this reduced game and check again whether the winner pays more than the expected value.

Solutions

Solution to Question 1. An ordinal game in strategic form is a list

$$\langle I, (S_1, \ldots, S_n), O, f, (\succsim_1, \ldots, \succsim_n) \rangle,$$

where:

- $I = \{1, 2, \ldots, n\}$ is the set of players;
- $(S_1, S_2, \ldots, S_n)$ is the list of strategy sets;
- O is the set of outcomes;
- $f : S \to O$ maps each strategy profile $s \in S = S_1 \times \cdots \times S_n$ to an outcome $f(s) \in O$;
- for each player i, $\succsim_i$ is a preference over outcomes.

Solution to Question 2.

(a) **Uncertain.** The statement concerns equilibrium existence, but some strategy profiles lead to lotteries and only ordinal preferences over outcomes are provided. Without knowing preferences over lotteries one cannot determine equilibrium existence from the given data alone.

(b) **True.** The two Bob nodes connected by one information set cannot start a subgame. The only proper subgame starts at the singleton Bob node on the right branch. Therefore subgames are: (i) the whole game, and (ii) that right-branch proper subgame.

Solution to Question 3.

(1) For Player 1, strategy C is strictly dominated by the mixed strategy

$$\sigma_1 = \begin{pmatrix} A & B & C \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix},$$

against L, M, R, this mixture gives payoff $1, 1, 1$, while C gives $0, 0, 0$. Eliminate C.

In the reduced game $\{A, B\} \times \{L, M, R\}$, Player 2 strategy R is strictly dominated by M.

The game reduces to:

	L	M
A	$(2, 1)$	$(0, 2)$
B	$(0, 2)$	$(2, 1)$

(2) No pure Nash equilibria. Let Player 1 play A with probability p, Player 2 play L with probability q. Indifference conditions are:

$$2q = 2(1 - q) \Rightarrow q = \tfrac{1}{2}, \qquad (2 - p) = (1 + p) \Rightarrow p = \tfrac{1}{2}.$$

Hence the unique Nash equilibrium is

$$\sigma = (\sigma_1, \sigma_2), \quad \sigma_1 = \begin{pmatrix} A & B & C \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} L & M & R \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}.$$

Solution to Question 4.

(1) Preference-contingent optimal return after S:

- **Generous type** ($U_B = m_B + 2m_A$):

$$U_B = (30 - b) + 2b = 30 + b,$$

so U_B is increasing in b, hence $b_G^* = 30$.

- **Selfish type** ($U_B = m_B$):

$$U_B = 30 - b,$$

so U_B is decreasing in b, hence $b_S^* = 0$.

(2) Ann's expected payoff:

$$\mathbb{E}[m_A \mid S] = p \cdot 30 + (1 - p) \cdot 0 = 30p, \qquad m_A(K) = 10.$$

(3) Ann chooses S iff

$$30p > 10 \iff p > \tfrac{1}{3}.$$

Therefore the threshold is

$$p^* = \tfrac{1}{3}.$$

At $p = \frac{1}{3}$, Ann is indifferent between K and S.

(4) Interpretation: trust emerges only if Ann believes the probability of meeting a generous Bob is sufficiently high. Here she must believe that at least one third of potential partners are generous.

Solution to Question 5.

(1) Each firm's expected value is

$$\mathbb{E}[V] = \tfrac{1}{2} \cdot 100 + \tfrac{1}{2} \cdot 0 = 50.$$

(2) If a firm wins with bid b, its expected payoff is $50 - b$. If both bid b, each wins with probability $\frac{1}{2}$ and gets $\frac{1}{2}(50 - b)$. The expected-payoff matrix is:

	40	60	80
40	$(5,\ 5)$	$(0,\ -10)$	$(0,\ -30)$
60	$(-10,\ 0)$	$(-5,\ -5)$	$(0,\ -30)$
80	$(-30,\ 0)$	$(-30,\ 0)$	$(-15,\ -15)$

(3) Best responses:

- Against 40: best response is 40 (payoff $5 > -10 > -30$).
- Against 60: best response is 40 (payoff $0 > -5 > -30$).
- Against 80: best response is 40 (payoff $0 = 0 > -15$); 60 is also a best response.

By symmetry the same holds for both firms. The unique pure-strategy Nash equilibrium is $(40, 40)$.

(4) At $(40, 40)$ each firm wins with probability $\frac{1}{2}$ and pays 40. Since $40 < \mathbb{E}[V] = 50$, the winner does *not* pay more than the expected value. Firms bid conservatively precisely to avoid overpaying.

(5) In the reduced game $\{60, 80\}$:

	60	80
60	$(-5,\ -5)$	$(0,\ -30)$
80	$(-30,\ 0)$	$(-15,\ -15)$

Best response to 60 is 60; best response to 80 is 60. The unique pure-strategy equilibrium is $(60, 60)$. The winner pays $60 > 50 = \mathbb{E}[V]$.

Comment. When firms can bid below the expected value, competition drives bids down and the curse is avoided. Removing the low bid forces both firms into bids that exceed the expected value, and the winner necessarily overpays. This illustrates how restricted competition or bidding floors can induce the winner's curse even when firms would otherwise avoid it.