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Game Theory - Spring 2026

Public Good Game

A group of $n \geq 2$ students is preparing a shared project for a game theory class they have together. Students are indexed by $i \in N = \{1, 2, \ldots, n\}$. Each student has an effort endowment of $e > 0$ (think of hours of effort or energies that could be dedicated to other tasks) and simultaneously chooses a contribution $c_i \in [0, e]$ to the common project.

Total group effort is

$$C = \sum_{i=1}^{n} c_i.$$

Student i's payoff is

$$\pi_i(c_i, c_{-i}) = e - c_i + rC, \qquad \frac{1}{n} < r < 1.$$

Here, r is the marginal per-capita return: one extra unit contributed to the class project generates a benefit of r for each student.

1. Fix contributions of all students other than i, denoted by c_{-i}. Derive student i's best response.
2. Find the set of Nash equilibria of the game.
3. Find the contribution profile that maximizes total class welfare $\sum_{i=1}^{n} \pi_i$.
4. Compare the Nash-equilibrium outcome and the socially optimal outcome, and briefly interpret the gap in this classroom context.
5. Suppose student i has *preferences for universalisation*: rather than taking others' contributions as fixed, she asks "what if everyone does the same as I do?" Formally, student i chooses c_i to maximise

   $$\tilde{\pi}_i(c_i) = e - c_i + r \cdot nc_i = e + (rn - 1)c_i,$$

   imagining that every other student $j \neq i$ also contributes $c_j = c_i$. What does student i choose? How does this compare to the Nash equilibrium and the social optimum?

Solution For student i, write

$$\pi_i = e - c_i + r\Big(c_i + \sum_{j \neq i} c_j\Big) = e + (r-1)c_i + r\sum_{j \neq i} c_j.$$

Since $r < 1$, we have $r - 1 < 0$, so π_i is strictly decreasing in c_i. Hence the unique best response is

$$c_i^*(c_{-i}) = 0.$$

Therefore, since $c_i^* = 0$ is a *dominant strategy* for each student i—it is optimal regardless of what others contribute—the unique Nash equilibrium is

$$c_1^* = \cdots = c_n^* = 0.$$

Total welfare is

$$W(c) = \sum_{i=1}^{n} \pi_i = ne - \sum_{i=1}^{n} c_i + nr \sum_{i=1}^{n} c_i = ne + (nr - 1) \sum_{i=1}^{n} c_i.$$

Because $r > 1/n$, we have $nr - 1 > 0$, so welfare is increasing in total contributions. Thus the social optimum is

$$c_i^{\mathrm{SO}} = e \quad \text{for all } i.$$

Interpretation: this is a free-riding problem. Individually, each student prefers to keep their own endowment (private marginal return is $r < 1$); collectively, full contribution is best for the class (social marginal return is $nr > 1$).

If student i universalises her choice, she perceives her payoff as

$$\tilde{\pi}_i(c_i) = e - c_i + r \cdot nc_i = e + (rn - 1)c_i.$$

Since $r > 1/n$, we have $rn - 1 > 0$, so $\tilde{\pi}_i$ is strictly increasing in c_i. The universalising student therefore chooses

$$c_i^{\mathrm{U}} = e.$$

This coincides with the socially optimal contribution. Intuitively, the universalisation preference internalises the positive externality of contributing: by imagining that her own choice is adopted by the whole group, student i effectively perceives the *social* marginal return $nr > 1$ rather than the private one $r < 1$. The free-rider problem dissolves not because incentives change, but because the student's reasoning criterion does.

Preferences for universalisation have been introduced by Laffont (1975) and further studied by Alger and Weibull (2013) and Roemer (2019).

References

Alger, I., & Weibull, J. W. (2013). Homo moralis—preference evolution under incomplete information and assortative matching. *Econometrica*, *81*(6), 2269–2302.

Laffont, J.-J. (1975). Macroeconomic constraints, economic efficiency and ethics: An introduction to kantian economics. *Economica*, *42*(168), 430–437.

Roemer, J. E. (2019). *How we cooperate*. Yale University Press.