TD7

1 Review Questions

---

e. In the model of the Easter Island seen in class, there is a threshold amount of resource such that population is growing above that threshold, and decreasing below that threshold.

- *Answer*

  True: You can easily see it from the phase diagram. We have a line along which S is fixed, above that line it increases and below it decreases. The expression of that line is $S = \frac{d-b}{\phi\alpha\beta}$, therefore if S is greater than $\frac{d-b}{\phi\alpha\beta}$ the population is growing, otherwise it is decreasing.

f. In the model of the Easter Island seen in class, the net growth of the resource (i.e. net of harvesting by humans) is decreasing in the numbers of humans present in the ecosystem, for a given value of the amount of the resource.

- *Answer*

  True: The net growth is given by the replenish of the resource minus the harvest, which is the part affected by the number of humans (population).

  $$\dot{S}(t) = \underbrace{rS(t)\left(1 - \frac{S(t)}{K}\right)}_{\text{Natural growth}} - \underbrace{\alpha\beta S(t)L(t)}_{\text{Harvest}}$$

  We can clearly see that an increase in $L(t)$ decreases the net growth. If you want to be more precise its enough to take the derivative with respect to $L(t)$:

  $$\frac{\partial \dot{S}(t)}{\partial L(t)} = -\alpha\beta S(t) < 0$$

  What you get is how much growth is smaller after an infinitesimal change in $L(t)$.

5 Equilibrium on Easter Island

In this exercise, we just have to reason with the phase diagram to get the answers. In the rest of the solution, I indicate with dashed lines the old equations, while continuous lines represents the new ones after the variable changes. We assume we are in the nontrivial internal steady state.

a. What happens if r goes up? Show on the graph the change to each conditions and the approximate dynamic transition to the new equilibrium if the convergence (spiral node with cyclical convergence). Interpret in a few words.

Let's start by breaking down all the components of the relevant equations.

$$S^* = \underbrace{K}_{intercept} - \underbrace{\frac{K\alpha\beta}{r}}_{slope} L^*$$

$$S^* = \underbrace{\frac{d-b}{\phi\alpha\beta}}_{intercept}$$

We can see that r only appears in one of the two equations, namely $S^* = K - \frac{K\alpha\beta}{r} L^*$. Since r is positively affecting its slope, but not the intercept, we have to rotate it counter-clockwise (as r increases). The other equation is not affected.

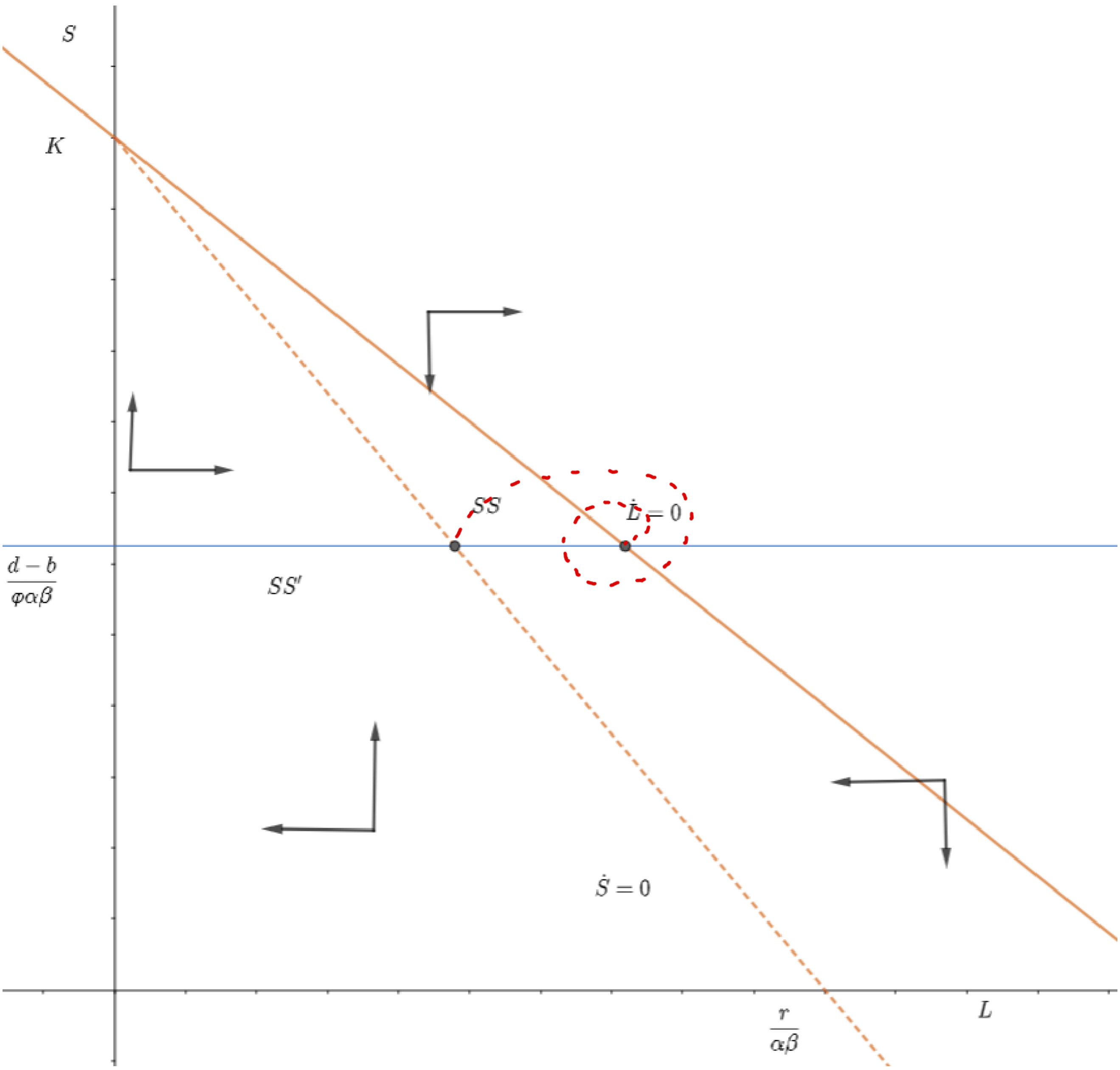

Figure 1: Phase diagram with new $r' = 0.06$.

What we see is that in the new steady state S^* is not affected, but L^* is higher. This is due to the fact that the increase of availability of resources is completely offset by the increase in population, which was possible because of the increase of the regeneration rate.

b. What happens if α goes up? Show on the graph the change to each conditions and the approximate dynamic transition to the new equilibrium if the convergence (spiral node with cyclical convergence). Interpret in a few words.

The rate α appears again in the intercept of the locus for $\dot{S} = 0$. However, in this case the rotation is clockwise as it is negatively affected by an increase in α. Moreover, α is also in the intercept for $\dot{L} = 0$, which goes down due to their negative relationship.

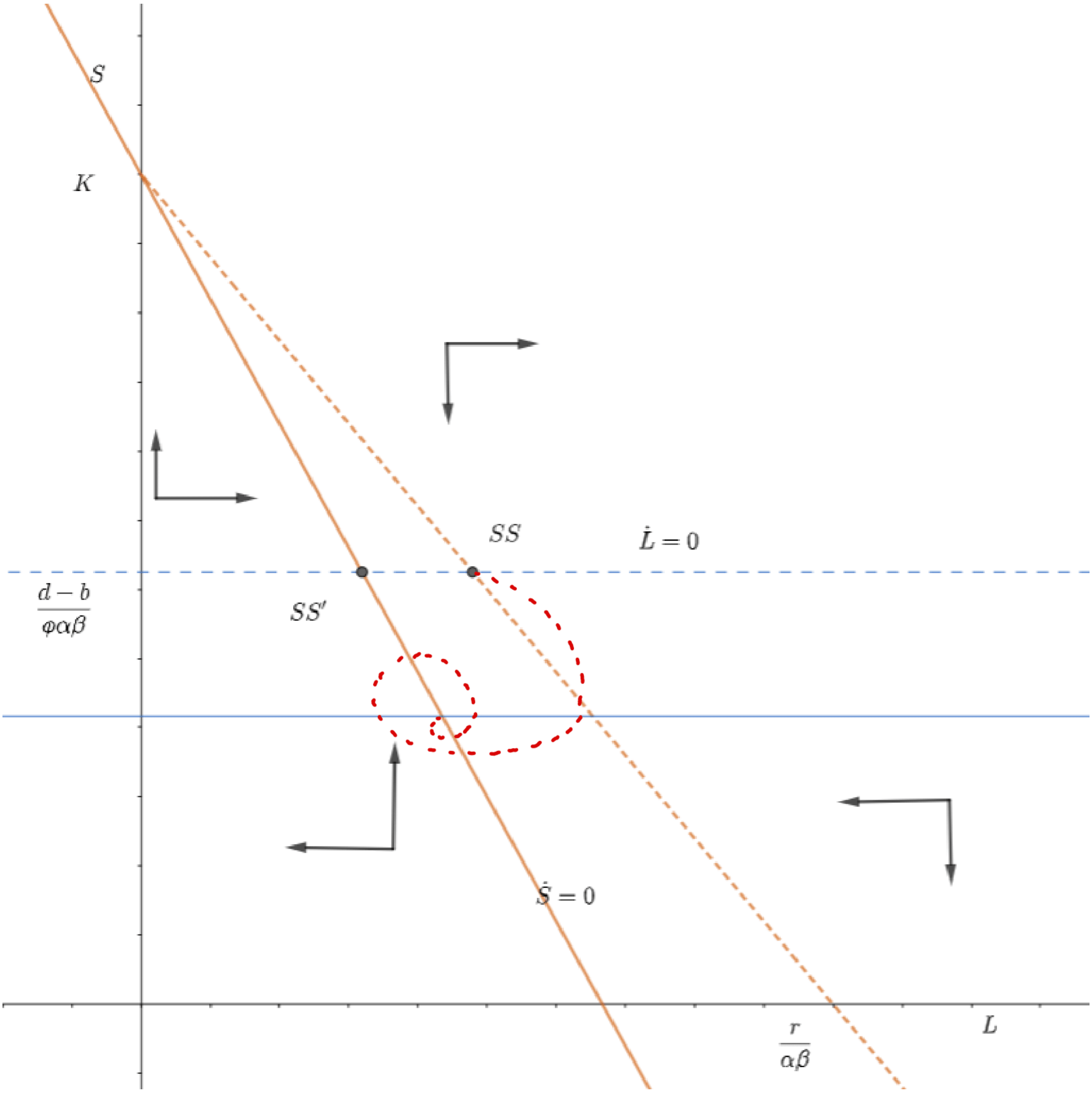

Figure 2: Phase diagram with new α' that leads to a decrease in L.

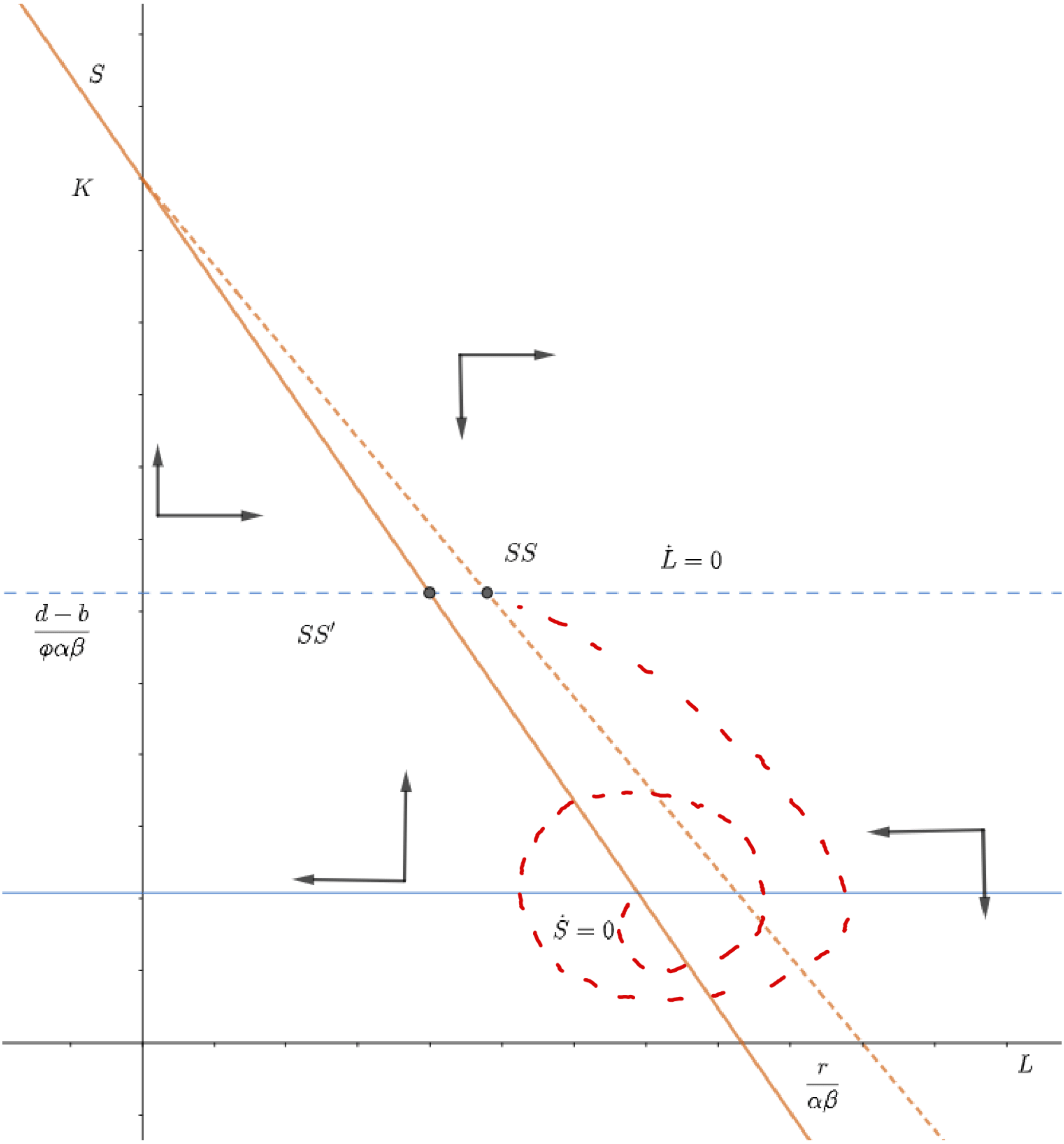

Figure 3: Phase diagram with new α' that leads to an increase in L.

In the new steady state we have unambiguously lower S^*. The impact on L^* depends on the entity of the increase of α, as illustrated by the pictures above. More efficient farming means that we need less workers to keep the amount of resources fixed (clockwise rotation). On the other hand, given the increase efficiency we need less resources to keep population constant (horizontal line decrease).

c. What happens if K goes down? Show on the graph the change to each conditions and the approximate dynamic transition to the new equilibrium if the convergence (spiral node with cyclical convergence). Interpret in a few words.

The capacity K only appears in the intercept and slope of $\dot{S} = 0$, which then must rotate counter clockwise.

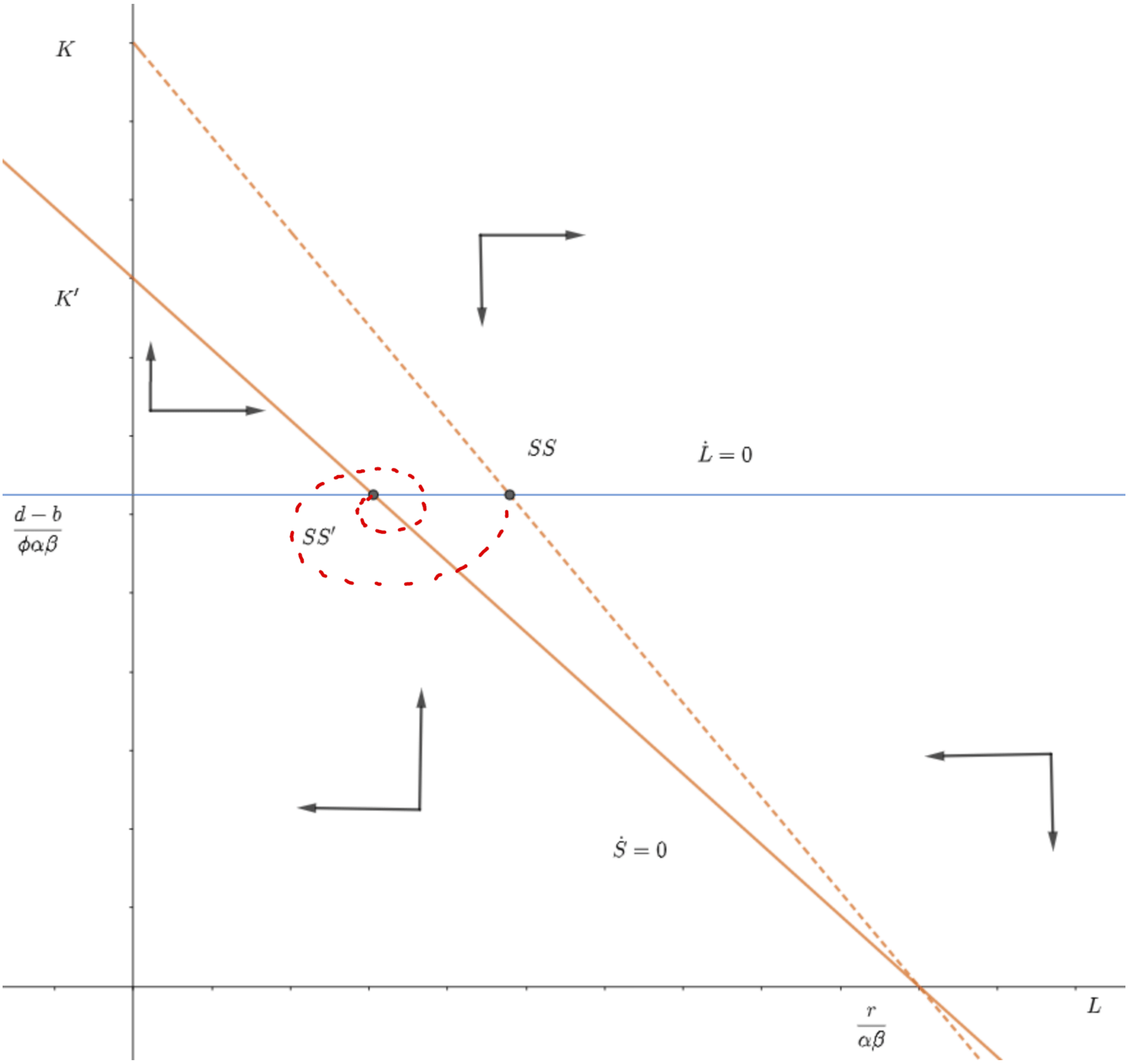

Figure 4: Phase diagram with new $K' = 9000$.

This time S^* is not affected while the impact on L^* is unambiguous. Due to the decrease in maximum capacity, the stock of resources grows slower. You can see this from the growth rate $\dot{S}$. This implies that L^* is lower than before.